# The ARCS Seminar

# Cartan pairs of algebras

Benjamin Steinberg

*City College of New York — CUNY Graduate Center*

**Abstract:** In the seventies, Feldman and Moores studied Cartan pairs of von Neumann algebras. These pairs consist of an algebra $A$ and a maximal commutative subalgebra $B$ with $B$ sitting "nicely" inside of $A$. They showed that all such pairs of algebras come from twisted groupoid algebras of quite special groupoids (in the measure theoretic category) and their commutative subalgebras of functions on the unit space, and that moreover the groupoid and twist were uniquely determined (up to equivalence). Kumjian and Renault developed the C*-algebra theory of Cartan pairs. Again, in this setting all Cartan pairs arise as twisted groupoid algebras, this time of effective etale groupoids, and again the groupoid and twist are unique (up to equivalence). In recent years, Matsumoto and Matui exploited that for directed graphs satisfying Condition (L), the corresponding graph C*-algebra and its commutative subalgebra of functions on the path space of the graph form a Cartan pair and exploited this to give C*-algebraic characterizations of continuous orbit equivalence and flow equivalence of shifts of finite type. The key point was translating these dynamical conditions into groupoid language. Since Leavitt path algebras are "Steinberg" algebras of the same groupoid, this led people to wonder about whether these dynamical invariants can be read off the pair consisting of the Leavitt path algebra and its subalgebra of locally constant maps on the path space. The answer is yes and it turns out in the algebraic setting, one doesn't even need Condition (L). Initially work was focused on recovering an ample groupoid from the pair consisting of its "Steinberg" algebra and the algebra of locally constant functions on the unit space. But no abstract theory of Cartan pairs existed and twists had not yet been considered. Our work develops the complete picture.

In this talk, we define a notion of an algebraic Cartan pair, and the more general notion of an algebraic quasi-Cartan pair. Historically, the first examples of quasi-Cartan pairs (although they were not termed this) were twisted group rings $R(G, c)$ over an integral domain $R$ having only trivial units. It has long been known that you can recover the group $G$ and the cohomology class of $c$ in this case from the isomorphism class of $R(G, c)$ as an $R$-algebra. We show that algebraic quasi-Cartan pairs arise precisely from twisted "Steinberg" algebras of ample groupoids that satisfy a groupoid analogue of the condition of having only trivial units. Moreover, the groupoid and twist are unique up to equivalence. For example, if you have an ample groupoid $\mathcal{G}$ all of whose isotropy groups are left orderable (like the case of path groupoids associated to directed graphs), then one gets a quasi-Cartan pair from any twist on $\mathcal{G}$.

This talk is based on joint work with: Becky Armstrong, Gilles G. de Castro, Lisa Orloff Clark, Kristin Courtney, Ying-Fen Lin, Kathryn McCormick, Jacqui Ramagge, and Aidan Sims

**Time and Place:** Wednesday, April 21 from 4:30–5:30PM (Mountain Time Zone) on Zoom. Contact Gene Abrams for the invitation link.