The ARCS Seminar

Conjugacy of integral matrices over algebraic extensions

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Abstract: The theory of matrix conjugacy is well-understood when all objects are defined over a field F, i.e., the matrices are taken to be in $F^{n\times n}$ and we conjugate by elements of $\mathrm{GL}_n(F)$. In restricting F to an integral domain, we find a more technical conjugacy theory with connections to number theory.

There is a concrete correspondence due to Latimer and MacDuffee (1933) between $\mathrm{GL}_n(\mathbb{Z})$-conjugacy classes of integral matrices with fixed irreducible characteristic polynomial and classes of fractional ideals. By this correspondence, we find that determining whether a pair of matrices are conjugate is equivalent to testing whether a particular ideal is principal.

In 2019, Marseglia generalized this correspondence to integral conjugacy classes of matrices with square-free characteristic polynomial and provided an algorithm for computing $\mathrm{GL}_n(\mathbb{Z})$-conjugacy classes of integral matrices.

I refer to the main problem in the talk as the conjugacy extension problem. This problem arises from an existence theorem of Guralnick, which states that integral matrices which are conjugate over the ring of p-adic integers for every prime p are conjugate over some algebraic extension of $\mathbb{Z}$. The conjugacy extension problem asks for the algebraic extension as in Guralnicks theorem.

We note that Marseglias correspondence for $\mathrm{GL}_n(\mathbb{Z})$-conjugacy can be extended to $\mathrm{GL}_n(R)$-conjugacy for any integral domain R. The generalization of the theory together with an adaptation of Marseglias algorithm allows us to test whether integral matrices are conjugate over a given algebraic extension of $\mathbb{Z}$. We also offer a method for searching for solutions to the conjugacy

extension problem which takes advantage of the principal ideal theorems of class field theory. We demonstrate that class fields do not provide a complete solution to (but are related to solving) the conjugacy extension problem in some examples.

Time and Place: Wednesday, March 16 from 4:30–5:30PM (Mountain Time Zone) in ENGR 187

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