The ARCS Seminar

On Naimark's Problem for Graph Algebras

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Abstract: This talk is dedicated to Professor Laszlo Fuchs on the occasion of his 99th birthday.

In 1951, M. A. Naimark proved that any two irreducible representations of the C^*-algebra $\mathcal{K}(\mathcal{H})$ of compact operators on a Hilbert space $\mathcal{H}$ are unitarily equivalent, and asked whether this property characterizes the algebra $\mathcal{K}(\mathcal{H})$ among C^*-algebras, that is, whether a C^*-algebra A with only one irreducible representation up to unitary equivalence is isomorphic to some $\mathcal{K}(\mathcal{H})$. Several researchers including Diximier, Kaplansky, Rosenberg produced initial partial solutions, but this problem remained unsolved until 2004 when C. Akemann and N. Weaver showed that Naimark's problem for an arbitrary C^*-algebra is undecidable in ZFC. So attention was focussed on considering Naimark's problem for special types of C^*-algebras. In 2017, N. Suri and M. Tomforde proved that Naimark's problem has a positive solution for a special type of graph C^*-algebra, called an AF algebra.

In this talk, we first give necessary and sufficient conditions on a graph E under which the corresponding graph C^*-algebra $C^*(E)$ is isomorphic to $\mathcal{K}(\mathcal{H})$. Using this, we extend the Suri-Tomforde theorem by showing that Naimark's problem has a positive solution for graph algebras of arbitrary graphs. We also characterize graph C^*-algebras $C^*(E)$ possessing exactly finitely many or exactly countably many irreducible representations no two of which are equivalent. If time permits, the algebraic version of Naimark's problem will be stated and proved for Leavitt path algebras.

Time and Place: Wednesday, Sept. 20 from 3:30–4:30PM (Mountain Time Zone) in ENG 187

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